

# Landau versus Spin Superfluidity in Two Component Bose-Einstein Condensates

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We consider two component Bose-Einstein condensates prepared in a single spin projection. In that case, density and spin waves can be described as distinct elementary excitations. This gives rise to separate critical velocities for the breakdown of the motional and the spin superfluidity. We demonstrate this behavior considering the interaction of the condensate with both a mechanical and a magnetic defect and calculate the corresponding drag forces.

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*Introduction.* Superfluidity [1] is undoubtedly the most striking and famous phenomenon linked with the Bose-Einstein condensation (BEC), as it corresponds to regimes in which the condensate is stable against the creation of excitations. According to the Landau criterion, it occurs whenever the condensate propagates at a velocity smaller than the speed of its excitations [2]. The latter are density waves, or bogolons [3], characterized by a linear dispersion for long wavelengths and a density-dependent celerity of sound. When the internal spin degree of freedom of the particles is taken into account, the situation becomes even more interesting. Indeed, spinor BECs [4, 5], which are not restricted to atomic systems but also include other systems such as exciton-polaritons [6, 7] and magnons [8, 9], have already brought remarkable discoveries, such as the formation of topological defects, exotic phases and spin textures [10, 11]. The physics of spinor BECs becomes even more reach in the presence of spin-orbit interactions [12–16].

In general, the superfluidity criteria for spinor BECs are less trivial, since the critical velocity is not uniquely defined [4]. As a consequence, spin and density degrees of freedom are mixed and the channel associated with pure spin excitations [17–19], in analogy to magnetism [20], is harder to identify. Therefore, the task of isolating, protecting, and controlling pure spin excitations can be of great interest for applications. The study of such systems leads to the understanding of spin superfluidity as a phenomenon distinct from the usual density (Landau) superfluidity: the onset of a spin-polarized (or magnetized) flow without spin wave excitation. In fact, spinor BECs possess undeniable similarities with magnetic systems: depending on the relative strength between the intra and inter-component interactions (tunable e.g. via Feshbach resonance [21]), the system can exhibit either ferromagnetic [22], anti-ferromagnetic [23], paramagnetic [24] or even diamagnetic features [25].

In this Letter, we investigate the superfluid properties of a magnetized spin-1/2 BECs [7, 17, 26]. The magnetization can be obtained (i) from a spin-anisotropic condensate prepared or naturally forming in a single spin component, or (ii) from a spin-isotropic BEC in the pres-

ence of an external magnetic field. By showing that the spin and density excitations can be completely separated, we argue that a spin current, protected against spin excitations, is possible for a wide range of parameters and even in the supersonic regime, where density excitations lead to the superfluidity breakdown. This suggests that a pure spin superfluid can be regarded independently from the usual motional (Landau) superfluid. We test the spin-superfluidity criterion by simulating the formation and suppression of spin waves in a condensate past a magnetic defect and calculate the corresponding drag force.

*Spin-1/2 condensate.* We consider a spinor BEC with two spin projections  $\pm 1$  ( $\pm$  indices) on the  $z$  quantization axis. At the mean field level, the system is governed by the energy functional

$$E = \int d\mathbf{r} \left[ \sum_{\sigma=\pm} \psi_{\sigma}^* \left( -\frac{\hbar^2 \Delta}{2m} + \frac{\alpha_1}{2} |\psi_{\sigma}|^2 \right) \psi_{\sigma} + \alpha_2 |\psi_+|^2 |\psi_-|^2 - \mathbf{H} \cdot \mathbf{S} \right] \quad (1)$$

written for the spinor

$$\Psi(\mathbf{r}, t) = \begin{pmatrix} \psi_+(\mathbf{r}, t) \\ \psi_-(\mathbf{r}, t) \end{pmatrix} = \begin{pmatrix} \sqrt{n_+(\mathbf{r}, t)} e^{i\theta_+(\mathbf{r}, t)} \\ \sqrt{n_-(\mathbf{r}, t)} e^{i\theta_-(\mathbf{r}, t)} \end{pmatrix} \quad (2)$$

in the Madelung representation, where  $n_{\pm}$  and  $\theta_{\pm}$  respectively represent the density and phase of each spin component. Here,  $\mathbf{H} = (H_x, H_y, H_z)^T$  is a generic (effective) magnetic field. The pseudospin vector  $\mathbf{S}$  allows the mapping to a magnetic system [27] and its components are linked to the spinor  $\Psi$  via the identities

$$\begin{aligned} S_x &= \frac{1}{2} (\psi_+ \psi_-^* + \psi_+^* \psi_-) = \sqrt{n_+ n_-} \cos(\Delta\theta) \\ S_y &= \frac{i}{2} (\psi_- \psi_+^* - \psi_-^* \psi_+) = \sqrt{n_+ n_-} \sin(\Delta\theta) \\ S_z &= \frac{1}{2} (n_+ - n_-), \end{aligned} \quad (3)$$

where  $\Delta\theta = \theta_+ - \theta_-$  is the relative phase and  $\alpha_{1,2}$  are the intra and intercomponent interactions. The  $S_z$  projection can be seen as the magnetization of the system.

The free energy of the condensate reads

$$F = -\mu n + (\alpha_1 + \alpha_2) \frac{n^2}{4} - H_x S_x - H_y S_y - (H_z - \Delta\alpha S_z) S_z, \quad (4)$$

where  $n = n_+ + n_-$ ,  $\Delta\alpha = \alpha_1 - \alpha_2$  and  $\mu$  is the chemical potential. The last term contains the applied magnetic field  $H_z$  and an intrinsic Zeeman splitting  $H_{Ze} = -\Delta\alpha S_z$  that embodies the spin-spin interaction. The ground state can be found from the minimization of  $F$ , at fixed total density  $n$ , with respect to the pseudospin components and given the normalization condition  $S_x^2 + S_y^2 + S_z^2 = n^2/4$ . Analytic solution can be found for the ground state for any  $\mathbf{H}$  but the expressions are cumbersome and it is more instructive to analyze particular cases.

For  $H_z = 0$  and  $H_{\parallel} = (H_x, H_y) \neq 0$ , there are two possible solutions. The first one is trivial  $(S_x^0, S_y^0, S_z^0)^T = (nH_x/2H_{\parallel}, nH_y/2H_{\parallel}, 0)^T$  and corresponds to a paramagnetic condensate having its pseudospin aligned with the in-plane field  $\mathbf{H}_{\parallel} = (H_x, H_y)^T$ . For a nontrivial solution with  $S_z \neq 0$ , which is not the ground state, we obtain

$$S_{x,y}^0 = \frac{H_{x,y}}{\Delta\alpha}, \quad S_z^0 = \pm \sqrt{\frac{n^2}{4} - \frac{H_{\parallel}^2}{\Delta\alpha^2}}, \quad (5)$$

possible for a spin-anisotropic condensate with  $\Delta\alpha \neq 0$ . The pseudospin is aligned with the *total* effective magnetic field  $\mathbf{H}_t = (H_x, H_y, -\Delta\alpha S_z)^T$ . Interestingly, the intrinsic field  $\mathbf{H}_{Ze} = -\Delta\alpha S_z \mathbf{u}_z$  attributes the condensate a *spin stiffness*, preventing the action of  $\mathbf{H}_{\parallel}$ . We see from Eq.(5) that  $S_z^0$  is characterized by a critical in-plane field  $H_c = \pm\Delta\alpha n/2$  below which the magnetization of the condensate would break down towards the trivial solution with  $S_z = 0$ . The chemical potential is found minimizing  $F$  with respect to the total density  $n$  giving

$$S_z = 0 \Rightarrow \mu = (\alpha_1 + \alpha_2) \frac{n}{2} - H_{\parallel}, \\ S_z \neq 0 \Rightarrow \mu = \alpha_1 n. \quad (6)$$

For  $H_z \neq 0$  and  $H_{x,y} = 0$ , two solutions are allowed as well. The trivial solution corresponds to a magnetized condensate with  $(S_x^0, S_y^0, S_z^0)^T = (0, 0, \pm n/2)^T$ . If  $S_{\parallel} \neq 0$ , we obtain

$$S_{\parallel}^0 = \sqrt{\frac{n^2}{4} - \frac{H_z^2}{\Delta\alpha^2}}, \quad S_z^0 = -\frac{H_z}{\Delta\alpha}. \quad (7)$$

As we can see from Eq.(7), for a given value of  $H_z$  the magnetization  $S_z^0$  can change its direction depending on the sign of  $\Delta\alpha$ : if  $\Delta\alpha > 0$ ,  $S_z$  is opposed to  $\mathbf{H} = H_z \mathbf{u}_z$  and the condensate behaves as an anti-ferromagnet; instead, if  $\Delta\alpha < 0$  the latter corresponds to a ferromagnet. Due to the intrinsic field  $H_{Ze}$ , the magnetization imposed by  $H_z > H_c$  would remain even after switching off the

applied field. On the other hand, the chemical potential reads

$$S_{\parallel} = 0 \Rightarrow \mu = \alpha_1 n - H_z \\ S_{\parallel} \neq 0 \Rightarrow \mu = (\alpha_1 + \alpha_2) \frac{n}{2}. \quad (8)$$

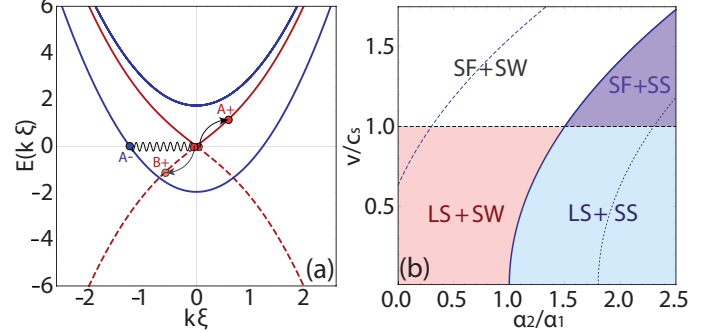


FIG. 1: (Color online) (a) Dispersion of the elementary excitations of the magnetized condensate. The solid (dashed) red line is the positive (negative) Bogoliubov branch of the condensed component and the arrows point the formation of a Bogolon. The dispersion of the non-condensed component (solid blue parabolas) is shifted down for  $H_z < H_c$  or up for  $H_z > H_c$ . The wavy line shows the excitation of a plane wave, allowed only in the former case. (b) Zero temperature phase diagram. SF: Supersonic Flow, SW: Spin Waves formation, SS: Spin Superfluid and LS: Landau Superfluid. The dashed, solid and dotted blue lines mark the frontier  $v = c_m$  in the cases  $H_z = \{-0.8, 0, 1.2\} mc_s^2$  respectively. The horizontal dashed line stands for  $v = c_s$ .

*Density and spin waves.* Let us now focus especially on the case where  $H_{x,y} = 0$ , treated as a perturbation in the following, and consider the elementary excitations on top of the fully magnetized condensate  $(S_x^0, S_y^0, S_z^0)^T = (0, 0, +n_0/2)^T$ . The minimization of the energy functional  $E$  leads to the following set of coupled Gross-Pitaevskii equations:

$$i\hbar \frac{\partial \psi_+}{\partial t} = -\frac{\hbar^2}{2m} \Delta \psi_+ - \mu \psi_+ + \alpha_1 |\psi_+|^2 \psi_+ \\ + \alpha_2 |\psi_-|^2 \psi_+ - \frac{H_z}{2} \psi_+, \quad (9)$$

$$i\hbar \frac{\partial \psi_-}{\partial t} = -\frac{\hbar^2}{2m} \Delta \psi_- - \mu \psi_- + \alpha_1 |\psi_-|^2 \psi_- \\ + \alpha_2 |\psi_+|^2 \psi_- + \frac{H_z}{2} \psi_-. \quad (10)$$

We start from the Bogoliubov ansatz [3]

$$\psi_+ = \sqrt{n_0} + A_+ e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} + B_+^* e^{-i(\mathbf{k} \cdot \mathbf{r} - \omega^* t)} \\ \psi_- = A_- e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}, \quad (11)$$

that involves two counter-propagating plane waves in the condensate characterized by the weak amplitudes  $A_+$  and  $B_+$ , the wavevector  $\mathbf{k}$  and frequency  $\omega$ . We seek

for a plane wave solution of amplitude  $A_-$  in the non-condensed component. Injecting (11) into Eqs. (9,10), we obtain the chemical potential  $\mu = \alpha_1 n_0 - H_z$  and the following modes

$$\hbar\omega_+^{(1,2)} = \pm\sqrt{E(k)(E(k) + 2\mu)} \quad (12)$$

$$\hbar\omega_- = E(k) + (2H_z - \Delta\alpha n_0), \quad (13)$$

where  $E(k) = \hbar^2 k^2 / 2m$ . The elementary excitations on top of the condensed component are the well-known density waves (or bogolons) following the dispersion  $\omega_{\pm} = \pm c_s k$  in the long wavelength limit, with  $c_s = \sqrt{\mu/m}$  being the speed of sound. The static condensate is stable against the creation of these excitations, that have a positive ( $|A_+|^2 - |B_+|^2$ )  $\hbar\omega_+^{(1)}$  contribution to the energy, and is therefore superfluid in the Landau picture. For the other spin component, we obtain a parabola shifted by a quantity  $2H_z - \Delta\alpha n_0$  due to the presence of the condensate (see Fig.1(a)). The coefficients  $A_+$  and  $B_+$  can be explicitly found provided the normalization condition  $|A_+|^2 + |B_+|^2 + |A_-|^2 = 1$ , thus yielding

$$(A_+^{(1,2)}, B_+^{(1,2)}) = \frac{1}{\sqrt{\mu^2 + \Delta E^2}}(\mu, \mp \Delta E), \quad (14)$$

where  $\Delta E = E(k) + \mu - \hbar\omega_+^{(1)}$  and  $A_-^{(1,2)} = 0$ . We then observe that the excitation of plane waves is forbidden unless the parabolic branch (13) is redshifted with respect to the condensate energy. Excitation (by a spin-coupling perturbation) of a plane wave  $\delta\psi_-(x, t) = A_- \exp(i\mathbf{k} \cdot \mathbf{r} - \omega t)$  leads to a spatio-temporal modulation of the relative phase  $\Delta\theta$ , leaving the condensate's density homogeneous. This kind of excitation can therefore be seen as a *spin wave*, in full analogy with magnetic systems [20], describing the spatial Larmor precession of  $\mathbf{S}$  about the  $z$ -axis. The corresponding deviation of the pseudospin follows from Eqs. (3-4), reading

$$\begin{aligned} \delta S_x &= \frac{A_-}{2} \cos(kx - \omega t), \\ \delta S_y &= \frac{A_-}{2} \sin(kx - \omega t), \quad \text{and} \\ \delta S_z &= \frac{n_0^2 - A_-^2}{2}. \end{aligned} \quad (15)$$

Interestingly, the transverse spin components  $\delta S_{x,y}$  oscillate out of phase, while the magnetization  $\delta S_z$  remains unchanged. This simply means that spin and density waves are decoupled. Moreover, the parabolic branch (13) is blue-shifted in respect to the anti-ferromagnetic condensate if  $H_z > H_c = \Delta\alpha n_0/2$ . The condensate is, in that case, stable against the creation of both density and spin waves, guaranteeing the simultaneous occurrence of *Landau* and *spin* superfluidity. We note that the ferromagnetic condensate automatically fulfills this requirement, even if  $H_z = 0$ .

To fully understand the physical meaning of spin superfluidity considered in this Letter, we should address the situation of condensate propagating with a velocity  $\mathbf{v}$ . In that case, the dispersions (12-13) are shifted by the quantity  $\hbar\mathbf{v} \cdot \mathbf{k}$ . According to the Landau's criterion, the superfluidity is lost for  $v > c_s = \sqrt{\mu/m}$  when the creation of bogolons becomes energetically favorable. In the same spirit, the formation of spin waves becomes possible as soon as the branch of spin waves acquires some negative energy (see insets of Fig.3), allowing the energy-conserving transfer from the  $\omega_+^{(1)}$  to  $\omega_-$  branches. Such process occurs above the critical velocity

$$c_m = \sqrt{\frac{4H_z - 2\Delta\alpha}{m}} = \sqrt{2}c_s \sqrt{\frac{\alpha_2}{\alpha_1} + \frac{H_z}{mc_s^2}} - 1. \quad (16)$$

The above expression allows us to construct the phase diagram shown in Fig.1(b), containing four regimes (see captions): exclusive Landau ( $c_m < v < c_s$ , pink region) or spin ( $c_s < v < c_m$ , purple region) superfluidity, the absolute superfluidity ( $v < c_s, c_m$ , light blue region) and the onset of both spin and density waves ( $v > c_s, c_m$ , white region).

*Generation of spin waves.* The most famous experiment to highlight superfluidity involves a condensate propagating against a potential barrier. As it is known, in the supersonic regime  $v > c_s$ , backscattered density waves appear upstream from the defect; in the subsonic regime, however, no radiation occurs.

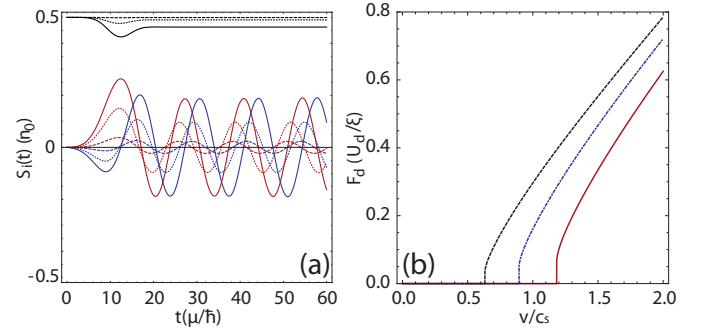


FIG. 2: (Color online) (a) Spin waves excited by a magnetic defect. The  $S_x$ ,  $S_y$  and  $S_z$  components are shown with red, blue and black lines respectively. The dashed, dotted and solid lines correspond to  $U_m/H_c = \{0.1, 0.4, 0.7\}$  respectively. (b) Spin drag force  $F_d$  experienced by the condensate. The dashed black, dashed dotted blue and solid red line corresponds to  $H_z = \{0, 1.4, 0.5\} mc_s^2$  and  $\alpha_2 = \{1.2, 0, 1.2\}$  respectively.

It is therefore natural to investigate what happens to the flow if a magnetic defect  $\mathbf{H}_x(\mathbf{r})$  is placed instead of a usual potential. For definiteness, we take  $H_z = 0$  and  $\Delta\alpha > 0$  in the following discussion. A first expected effect regards the conversion from the  $+$  to the  $-$  component via spin rotation. Naturally, the conversion efficiency depends on both the strength of the field and on

the speed of the flow. Second, beyond the defect, the residual magnetization  $H_{Ze}$  leads to self-induced Larmor precession of the pseudospin, thus creating a spin wave as discussed previously. To get some intuition we can consider a zero-dimensional system in which the magnetic defect  $H_x$  is switched on and off. In such configuration, the dynamics of the pseudospin can then be described by an undamped Landau-Lifshitz equation  $\hbar\partial_t\mathbf{S} = \mathbf{H} \times \mathbf{S}$ , which leads to the following set of nonlinear equations

$$\hbar\partial_t S_x = \Delta\alpha S_z S_y, \quad (17)$$

$$\hbar\partial_t S_y = (-\Delta\alpha S_x - H_x) S_z, \quad \text{and} \quad (18)$$

$$\hbar\partial_t S_z = H_x S_y, \quad (19)$$

to be solved with respect to the initial condition  $\mathbf{S}_0 = (0, 0, n_0/2)$ . Here, we assume that  $\mathbf{H}_x = H_x(t)\mathbf{u}_x$  with  $H_x(t) = U_m \exp[-(t - t_0)^2/\Delta t^2]$  representing a pulse of duration  $\Delta t$  switched on at the instant  $t_0$ . Eqs. (17-19) can be solved analytically in the subcritical regime  $U_m \ll H_{Ze}$ . Up to a small phase factor, the solution reads

$$S_x(t) = -\frac{\Delta t}{\Delta t_m} S_y^0 \sin(H_{Ze}^0 t) \quad (20)$$

$$S_y(t) = +\frac{\Delta t}{\Delta t_m} S_y^0 \cos(H_{Ze}^0 t) \quad (21)$$

$$S_z(t) = \sqrt{\frac{n^2}{4} - \left(\frac{\Delta t}{\Delta t_m} S_y^0\right)^2} \quad (22)$$

at long times, where  $H_{Ze}^0 = \Delta\alpha S_z^0$ ,  $\Delta t_m = \hbar\pi/\sqrt{U_m^2 + H_{Ze}^2}$  and  $S_{y,z}^0$  are given by Eq.(5). Full numerical solutions to Eqs. (17-19) are presented in the Fig.2(a), showing the dynamics of the pseudospin precession. We can observe the formation of out-of-phase oscillations of  $S_x$  and  $S_y$ . By increasing the value of  $U_m$ ,  $S_z$  (and, consequently,  $H_{Ze}$ ) is reduced, which in turn increases the magnitude and reduces the frequency of the spin wave as captured by Eqs.(20-22).

To further understand the differences between the usual and spin superfluidity, we address the question of dissipation in the spin flow. In the presence of a magnetic defect, the drag force is defined as  $\mathbf{F}_m = -\int(\psi_+^* \nabla H_x \psi_- + \text{c.c.})d\mathbf{r}$ . For simplicity, we consider a localized defect of the type  $H_x = U_m \delta(\mathbf{r})$ , such that  $\mathbf{F}_m = U_d \int(\psi_+ \nabla \psi_-|_{\mathbf{r}=0} + \text{c.c.})d\mathbf{r}$ . Expanding the wave functions according to (11), and using the Hopfield amplitudes (14) modified by the Doppler shift  $\hbar\mathbf{k} \cdot \mathbf{v}$ , we obtain, in first order in  $A_\pm$  and  $B_\pm$ ,  $\mathbf{F}_m = U_m n_0 \int i\mathbf{k} S_f(k, \mathbf{k} \cdot \mathbf{v}) d\mathbf{k}/(2\pi)^2$ , where

$$S_f(k, \omega) = \frac{1}{2m} \frac{k^2}{(\omega + i0)^2 - \omega_-^2} \quad (23)$$

represents the dynamic structure factor associated with the spin excitations. The infinitesimal positive imaginary part  $+i0$  is added to the frequency  $\omega$  to be consistent with

the usual Landau causality prescription. Computing the integral of (23) in the complex plane by means of the relation  $1/z + i0 = P(1/z) + i\pi\delta(z)$ , and noticing that the integral over  $\varphi$  ( $\varphi$  is the angle between  $\mathbf{k}$  and  $\mathbf{v}$ ) vanishes for the principal part  $P$  as a consequence of symmetric integration limits, we easily obtain the explicit expression

$$\mathbf{F}_m = \frac{U_m n_0 m^2}{12\pi\hbar^3 v^2} \left(v + \sqrt{v^2 - c_m^2}\right)^3 \mathbf{u}_v. \quad (24)$$

We observe that the force is zero for  $v < c_m$  and monotonically increases with  $v$  if  $v > c_m$  (see Fig.2(b)). For overcritical flows,  $v \gg c_m$ , the force reduces to the classical result  $F_m \propto v$ . Notice that Eq. (24) is formally similar, but physically different, to the drag force  $\mathbf{F}_d = \alpha_d^2 n_0 m^2 (v^2 - c_s^2)/(\hbar^3 v) \mathbf{u}_v$  acting on the superfluid density in the presence of a localized defect  $V_d = \alpha_d \delta(\mathbf{r})$  [28], where the role of critical velocity  $c_m$  is played by  $c_s$ .

*Numerical results.* To illustrate the results discussed previously, we performed numerical solutions of Eqs. (9,10) for parameters corresponding to different regions of phase diagram depicted in Fig.1(b). We considered a 2D homogeneous magnetized condensate propagating with the velocity  $\mathbf{v} = v_x \mathbf{u}_x$ , by setting the condition  $\Psi_0(\mathbf{r}) = (\sqrt{n_0} e^{ikx}, 0)^T$ , with  $k_x = m/\hbar v_x$ . In order to distinguish between the Landau and the spin superfluidity, we plugged a potential barrier and a magnetic defect, well separated from each other, to respectively excite density and spin waves in the flow. The latter demands an extra *local* coupling term  $-H_x(\mathbf{r})\psi_\mp/2$  in the equations. The results are summarized in Fig.3, where the four regions of the phase diagram are considered. We enclose snapshots of the linear wave function  $\psi_X = \psi_+ + \psi_-$ , which is the best suited quantity to illustrate density and spin waves simultaneously. In each panel, the insets show the corresponding dispersion of elementary excitations. Panel (a) corresponds to a supersonic flow configuration in which spin waves can develop as well ( $v > c_s, c_m$ ). Interestingly, while the density waves form upstream from the barrier, the spin waves are found downstream due to their positive group velocity. Panel (b) shows the exclusive spin superfluid regime in a supersonic flow, where only density excitations take place ( $c_s < v < c_m$ ). Panel (c) illustrates the exclusive Landau superfluidity, for which only spin waves can be excited ( $c_m < v < c_s$ ). Finally, panel (d) demonstrates the absolute superfluidity of the spinor condensate, in which no excitations of any kind are observed ( $v < c_s, c_m$ ).

*Conclusions.* We have shown that magnetized two-component condensates allow the formation of both density and spin waves that can be suppressed independently depending on the type of spin-spin interaction, the applied magnetic field, and the speed of the flow. In such system, superfluidity requires both the Landau and spin superfluidity criteria to be fulfilled. We have constructed the phase diagram of the system and illustrated the latter with a propagating condensate against a regular and



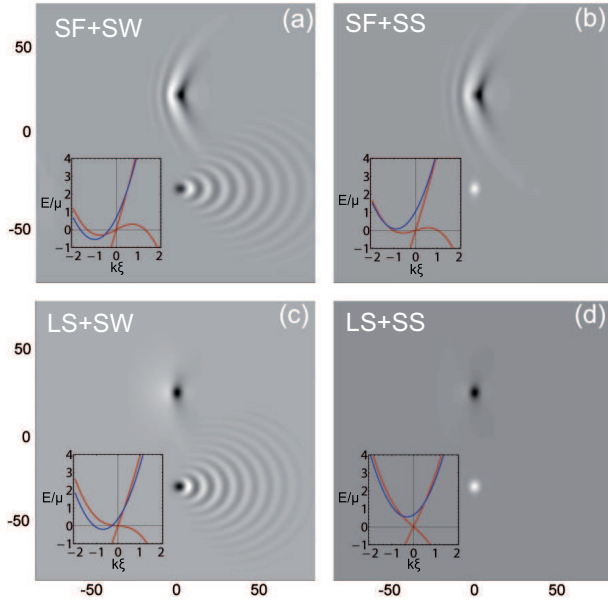


FIG. 3: (Color online) The four phase diagram regimes illustrated numerically with both a regular (upper) and a magnetic (lower) defect within the same flow. The colormap shows  $|\psi_X|^2$  and the insets display the dispersions. The visual order of the phases is preserved with respect to Fig.1(b).

magnetic defect and calculated the spin drag force. A well suited system to implement experimentally our proposals would be a circularly polarized exciton-polariton condensate either in ferromagnetic regime [29] or under an applied magnetic field to compensate for the antiferromagnetism [25, 30]. This system is to our knowledge the one offering to tune the parameters of a flowing condensate in the most flexible way [31]. Finally, the observation of superfluid propagation of spin currents would represent an important milestone in spintronics and spinoptronics.

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